Mathematics of Computing
Mid Semester (Max Marks 40, Time 2h)

Indian Statistical Institute, Bangalore

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Q1. ($1\times 10=10$) True or false:

i Complement of a regular language is regular.

ii The set difference between regular languages is regular.

iii The empty language is the one that accepts the empty string.

iv Every subset of a regular language is a regular language.

v Every finite set of strings is regular.

vi If $L_1 = \phi$ and L_2 is regular and not empty then $L_1 \cdot L_2$ is not empty.

vii The set of strings accepted by a DFA does not change if we add a self-loop with label ϵ to any state of the DFA.

viii A non-regular grammar always produces a non-regular language.

ix The set of strings on $\{0, 1\}$ *without* the substring "000" is regular.

x The language produced by the following productions is regular.
$S \to bBaA$
$A \to Aa|a$
$B \to bB|b$

Q2. (3+3+1+5=12)

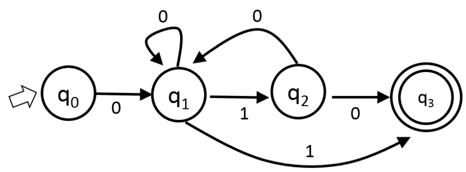
(a) State and prove the *pumping lemma* for regular languages.

(b) Prove that the language $L = \{ww^R | w \text{ is a binary string}\}$ is not regular. Here w^R represents the reverse of the string w.

(c) Languages A, B and C are such that $A = B \cap C$; C is regular. Show that if A is not regular then B is not regular.

(d) Consider the language L is the set of all strings of the form $a^i b^j c^k$ where if $i = 1$ then $j = k$. Show that L is not regular. (Hint: Prove that $L_1 = \{ab^n c^n | n \geq 0\}$ is not regular. Then use the result from part (c).)

Q3. (2+3+2+3=10) Consider the following NFA.



(a) Formally define the NFA in terms of the 5-tuple $(Q, \Sigma, \delta, q_0, F)$

(b) Convert the NFA to a DFA by constructing state subsets. Clearly drawing the DFA with state subsets suffices.

(c) Is your resulting DFA minimal or not? Justify.

(d) Use the GNFA method to convert the resulting DFA into a regular expression. In the process show the GNFA after each state elimination.

Q4. (2+2+2+2=8)

(a) Let $G = (\Sigma, V, S, \mathcal{P})$, $\Sigma = \{0, 1\}$, $V = \{S, A\}$, and $\mathcal{P} :=$
$P1 : S \to 0A$
$P2 : S \to 01$
$P3 : A \to S1$
Is G regular? If yes, write a regular expression for the language. If not, then what property of regular grammars is violated?

(b) Construct an NFA for the following regular grammar: $G = (\Sigma, V, S, \mathcal{P})$, $\Sigma = \{0, 1\}$, $V = \{S, A\}$, and $\mathcal{P} :=$
$P1 : S \to 11A|00B$
$P2 : A \to 01A|10A$
$P3 : A \to \epsilon$
$P4 : B \to 00B|11B$
$P5 : B \to \epsilon$

(c) Construct an NFA for the following regular expression:

$$(0|1)^*(00)^+(0|1)^*$$

(d) A student is asked to convert a DFA M_L for a language L to a DFA or NFA for the language L^*. He does the following two modifications to M_L : (1) mark the initial state as a final state; and (2) Add an ϵ transition from each final state to the initial state. Show by example that the construction is invalid. Give the valid construction.